Graph Theoretic Models for Ad hoc Wireless Networks

Prof. Srikrishnan Divakaran

DA-IICT

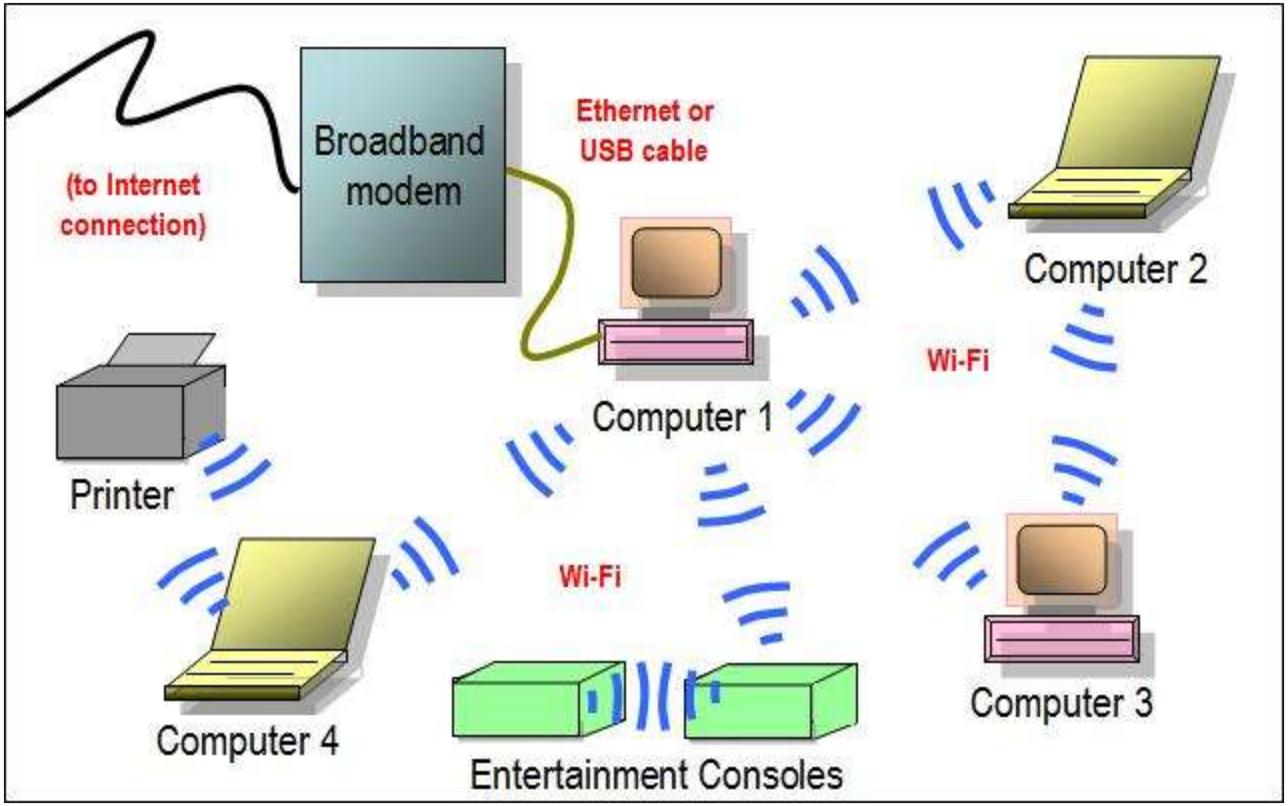
Talk Outline

- Overview of Ad hoc Networks
- Design Issues in Modeling Ad hoc Networks
- Graph Theoretic Models

Overview of Ad hoc Networks

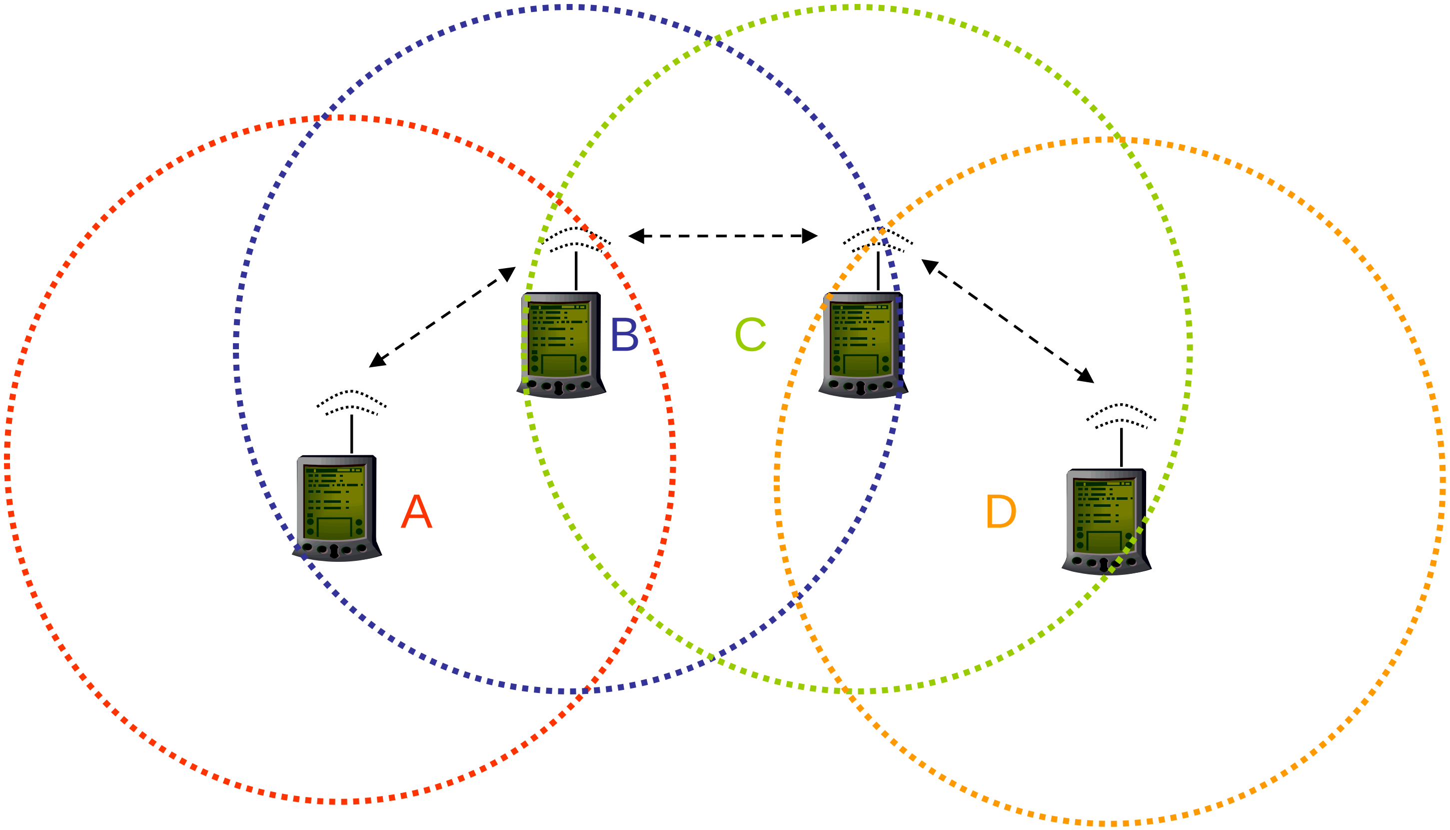
- Node characteristics
- Network characteristics

Ad hoc Networks (1)



An example of an Ad hoc Wireless Network

Characteristics of a node in an Ad hoc Network (1)

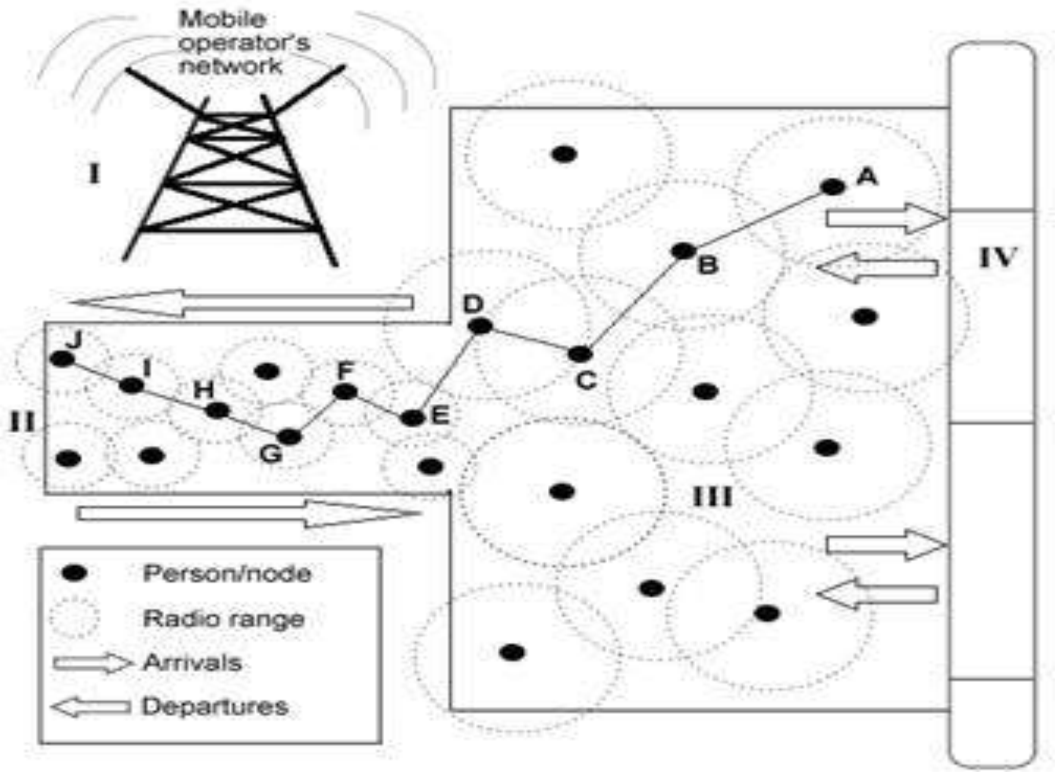


A set of nodes in an Ad hoc wireless network

Characteristics of a node in an Ad hoc Network (2)

- An Ad hoc wireless network consists of a set of nodes (wireless devices) where each node
  - has an unique identifier
  - has a finite communication range (i.e. radio range)
  - has a processor with a finite processing speed and cache
  - is mobile and capable of communicating directly with nodes that are within its communicating range and indirectly (i.e. through inter-mediate nodes) with nodes that are not within its communicating range
  - is constrained by battery power, consumes most of its power while sending packets and very little while receiving or processing packets

An Ad hoc Mobile Network



Ad hoc Network Characteristics (1)

- An Ad hoc wireless network
  - is self-organizing, adaptive and does not depend on any fixed network infrastructure
  - Typically consists of heterogeneous nodes (i.e. nodes with different processing speeds, storage and communication ranges)
  - Consists of communication links of finite bandwidth between nodes that are within each others communication range
  - Not all nodes are within the transmission range of every other node. So, nodes need to cooperate for forwarding packets

Ad hoc Network Characteristics (2)

- In an Ad hoc wireless network
  - node mobility causes frequent changes in network topology and limits the use of static routing protocols
  - lower link capacity and multiple hops lead to higher transmission delays and overheads
  - Unpredictable link properties leads to higher errors in transmission
  - Node mobility and autonomous behavior of nodes requires regular update of state information
  - Power consumption at the nodes places constraints on transmission

Design issues in Modeling

- Energy constraints
- Node mobility
- Network topology
- Type of protocol
- Type of control
- Optimality criteria

Energy Constraints (1)

- Wireless devices have limited battery life.
- Each node consumes power depending on whether it is
  - Asleep (i.e. no transmission)
  - Idle (i.e. listening but not sending)
  - transmitting (i.e. sending but blocking its channel).
- transmitting packets is considered more expensive than receiving or processing packets
- switching from asleep mode to idle mode is expensive
- Protocol architecture impacts the energy requirements
- Energy constraints restricts the topology of a network (i.e. hop count, degree of a node)

Energy Constraints (2)

- Goal: Choose an energy aware cost function that satisfies the energy constraints and is expressed in terms of
  - Initial energy,
  - current energy
  - unit transmission cost
- Cost Functions can be broadly classified as
  - Linear
  - Quadratic
  - Parameterized

Energy Constraints (3)

- Some examples of cost metrics are
  - Hop count or delay
  - energy consumed per packet
  - Time to partition network
  - variance in node energy levels.

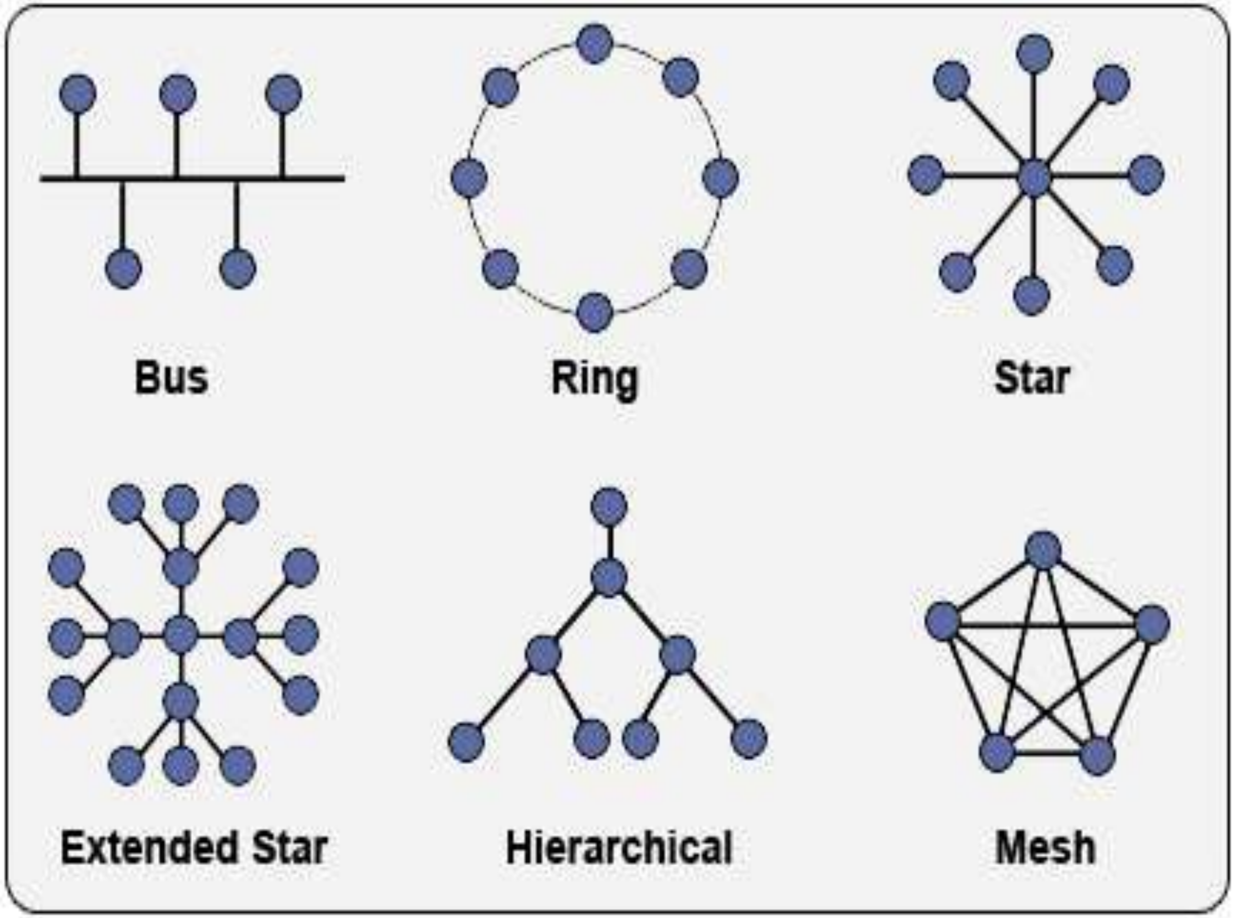
Node Mobility (1)

- Node mobility can cause frequent and unpredictable changes in network topology so static routing based protocols are of limited use
- Each node knows its position relative to its one hop neighbors in order to route packets
- A nodes neighborhood information may need to be updated periodically
- Address schemes based on location would be preferred

Node Mobility (2)

- Packets need to be forwarded in the geographical direction of the destination based on local information
- Distributed demand based localized routing more appropriate
- Routing should be loop free in order to be scalable
- Cost metrics can be a combination of loop count, power and distance
- Unit disk graphs can be used to model mobility by providing position based routing

Network Topology



Type of Protocol

- Proactive
  - Requires maintaining routing tables
  - Regular update of state is required
  - More efficient routes can be chosen
- Reactive
  - Routes are determined on demand basis
  - Less overhead but not always efficient
- Hybrid
  - Proactive within a local region but reactive across regions

Type of Control (1)

- Centralized
  - Global State needs to be maintained and shared
  - Higher overhead in periodically sharing state
  - Proactive protocols are appropriate
  - Routing Algorithms based on global information are likely to be optimal and are likely to be loop free
  - There will be scalability issues when there are frequent state changes and node mobility

Type of Control (2)

- Distributed
  - Local state may be sufficient so less overhead for sharing state information
  - Reactive protocols are appropriate
  - Greedy algorithms are appropriate
  - Routing algorithms based on local information may not be optimal and may not be loop free
  - appropriate for situations where the network topology is dynamic

Optimization Criteria

- Hard versus soft guarantees
- User versus System objectives
- Differentiation between Users
- Some examples of optimality criteria
  - Throughput, Response Time, Network Life, Jitter, Delay, Availability, Packet Loss, Hop Count, Energy Utilization.

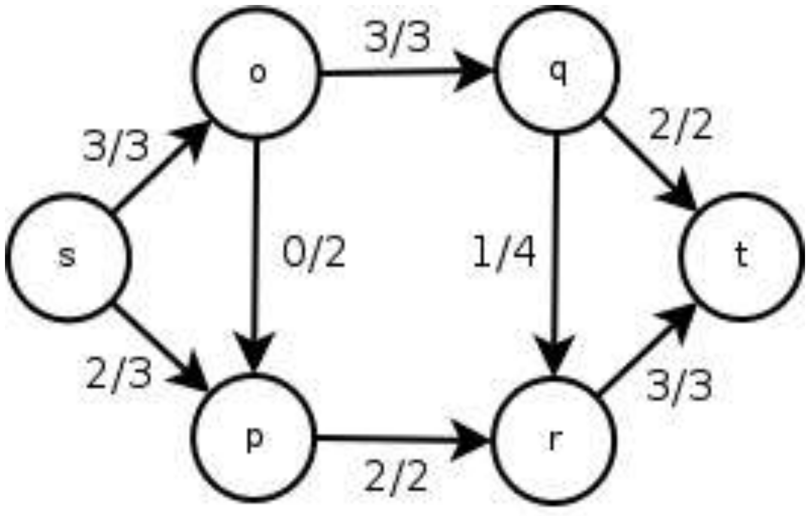
Graph Theoretic Models

- Minimum Cost Flow Models
  - Shortest path problem
  - Maximum flow problem
  - Assignment problem
  - Generalized flow problem
- Spanning Tree Models
  - Minimum spanning Tree problem
  - Steiner Tree problem
- Covering/Packing Models
  - Minimum dominating set
  - Unit Disk Graph
- Congestion Pricing Model
  - Virtual circuit routing

Minimum Cost Flow Problem

- Given a flow network G = (V, E), where
  - V is the set of n nodes
  - E is the set of m directed edges
  - s is the source node and t is the sink node
  - For each node i, b(i) denotes its supply/demand
  - For each edge (i, j) in N
    - c(i,j) – cost per unit flow on (i, j)
    - u(i,j) – maximum flow of flow on (i, j)
    - l(i,j) – minimum amount of flow on (i, j)
    - x(i, j) – the flow on (i, j) that needs to be determined

Minimum Cost Flow Problem



Minimize $\Sigma c(i,j)x(i,j)$

Subject to

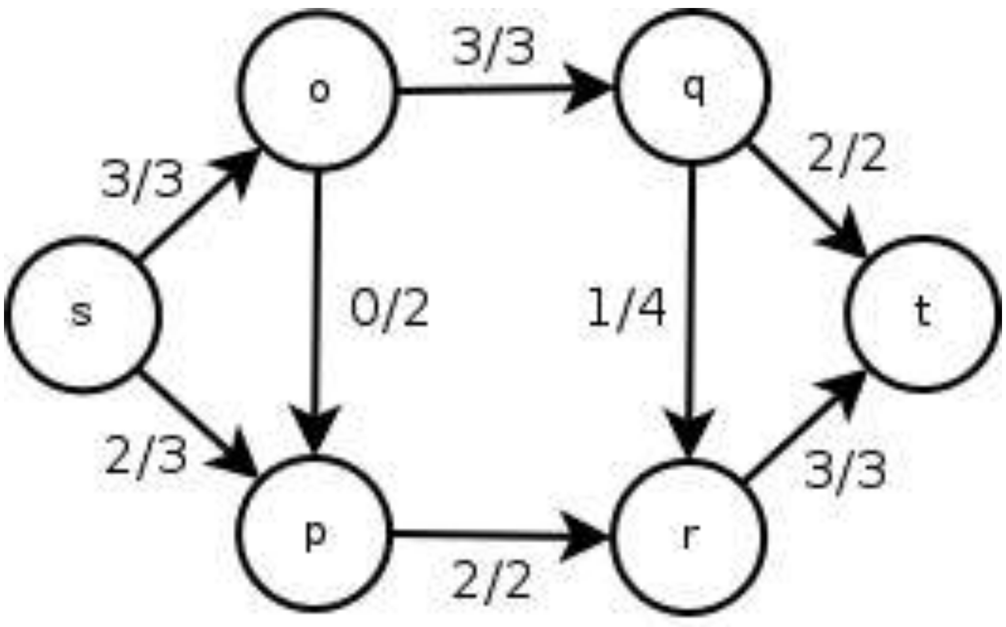
- Capacity constraints: $x(i,j) <= u(i,j)$
- Skew symmetry: $x(i,j) = -x(j,i)$
- Flow conservation: $\Sigma_j x(i,j) = 0$ for all $i \neq s, t$
- Required flow: $\Sigma_j x(s,j) = d$ and $\Sigma_j x(j,t) = d$

Shortest Path Problem

- Find the minimum length (cost) path from a specified source node s to another specified sink node t.
- A special case of minimum cost flow problem where
  - C(i,j) is the distance between the nodes i and j
  - There are no capacity constraints
  - Required flow: $\Sigma_j x(s,j) = \Sigma_j x(j,t) = 1$

Maximum Flow Problem (1)

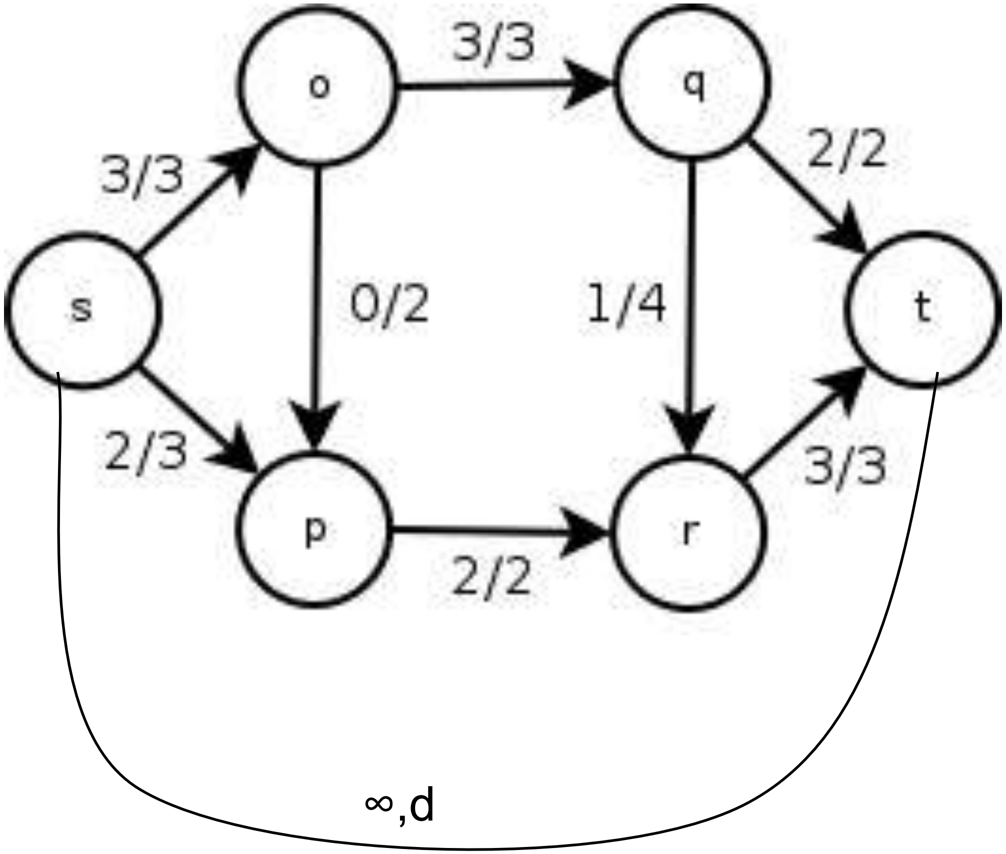
- Determine the maximum amount of flow from a specified source s to another specified sink node t



Maximum Flow Problem (2)

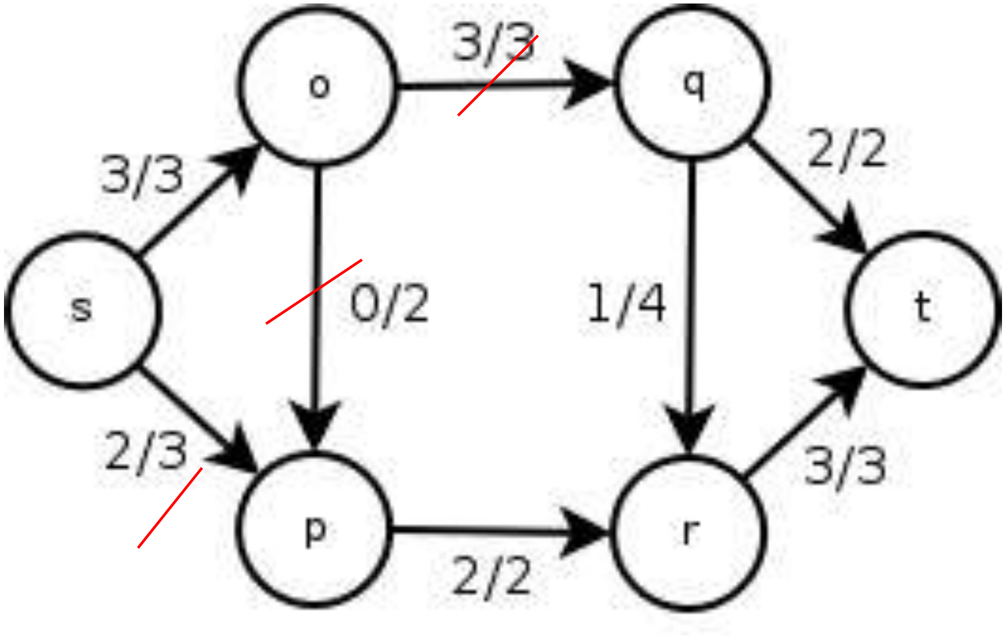
- Determine the maximum amount of flow from a specified source s to another specified sink node t
  - There are no costs associated with flows on arcs
  - U(i,j) – maximum capacity of the arc (i,j)
  - C(i,j) – set to 0 for all arcs in E
  - b(i) = 0 for all nodes in V
  - introduce an arc (t,s) with cost c(t,s) = -1 and set its capacity u(t,s) = ∞.

Maximum Flow Problem (3)



Minimum Cut Problem

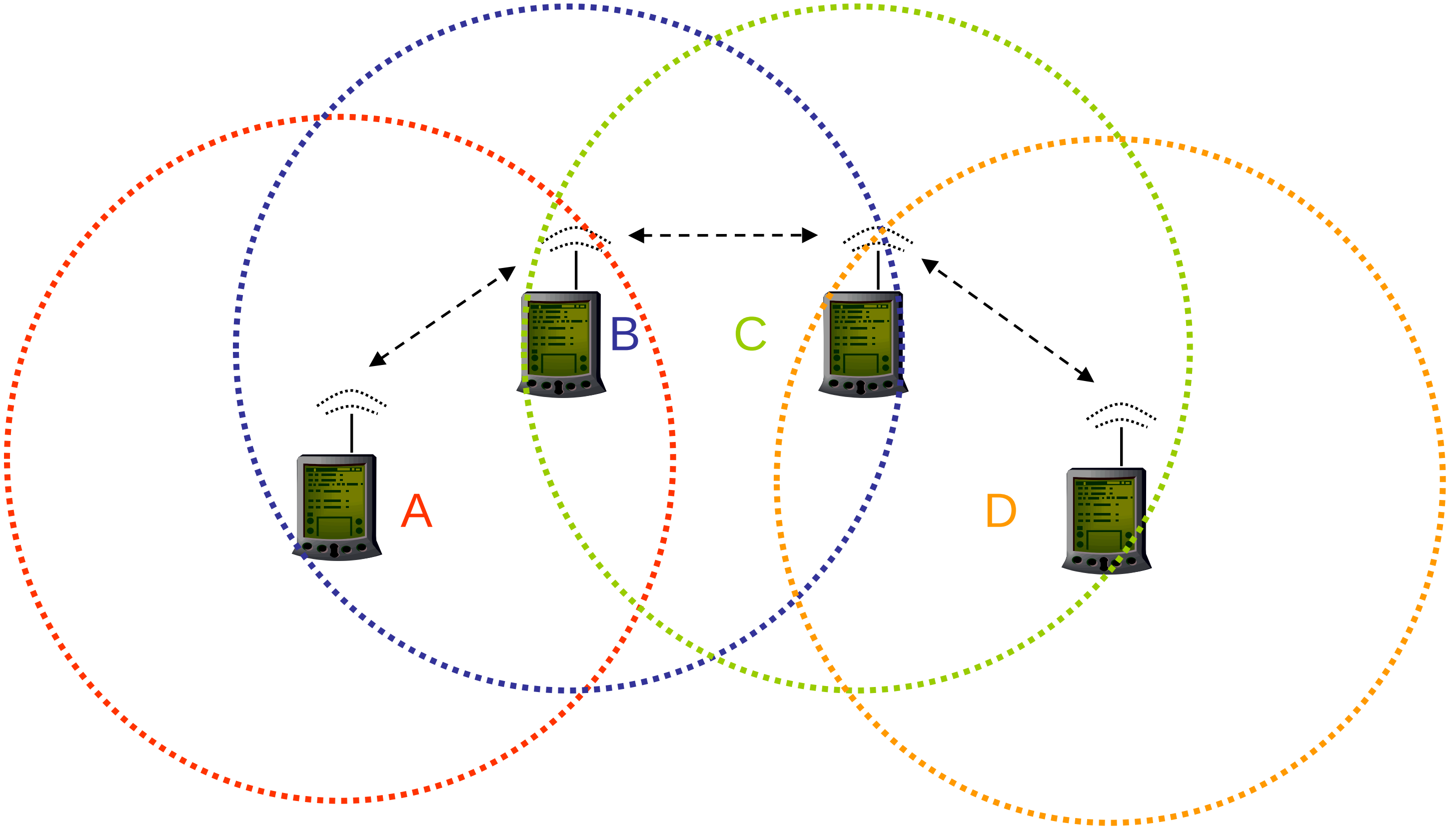
<u>The max-flow min-cut theorem</u>: **The maximum value of an s-t flow is equal to the minimum capacity of an s-t cut**.



Assignment Problem

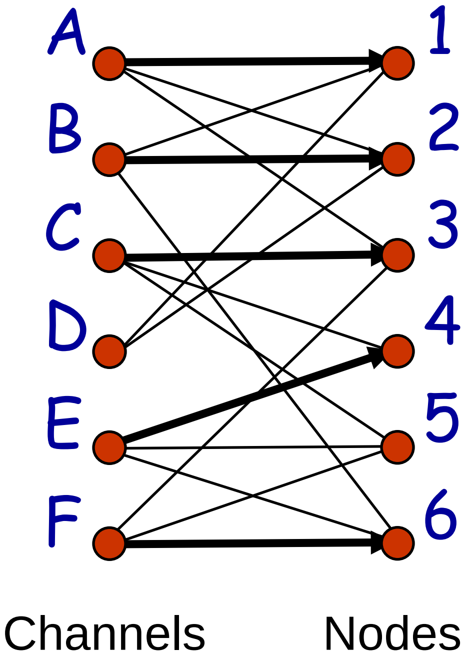
- Given two sets N_1 and N_2, a collection of pairs A in N_1 X N_2 representing possible assignments, and a cost c(i,j) associated with each arc (i, j), find a minimum cost assignment.
  - $G = (N_1 U N_2, A)$
  - $B(i) = 1$ for all i in N_1 and $b(i)=-1$ for all i in N_2
  - $U(i,j) = 1$ for all (i,j) in A

Assignment Problem



Assignment of Bandwidth to nodes to balance load

Restricted Assignment Problem

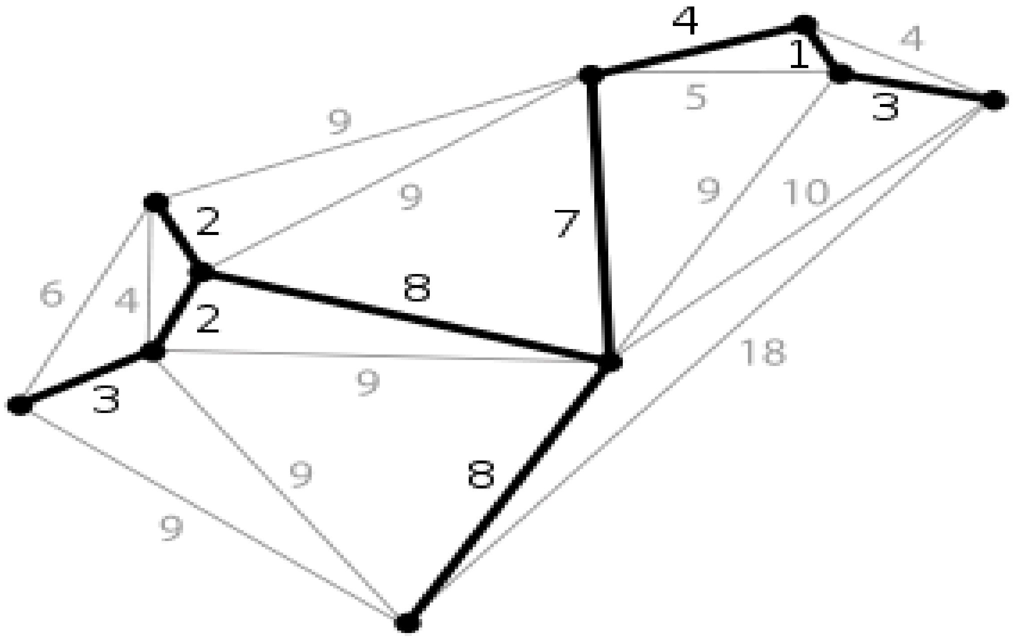


Assignment of Channels to nodes to maximize throughput

Generalized Flow Problem

- A generalization of minimum cost flow problem
- If x(i,j) units enter an arc (i,j) then σ(x(i,j)) units arrive at node j, where for
  - $0 < \sigma(x(i,j)) <= 1$ it is a lossy network
  - $\sigma(x(i,j)) > 1$ it a gainful network

Minimum Spanning Tree Problem (1)



An Example of a minimum spanning Tree

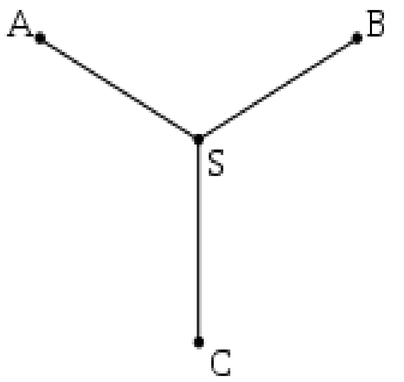
Minimum Spanning Tree Problem (2)

- A spanning Tree is a tree than spans all the nodes of an undirected network.
- The cost of the spanning tree is the sum of the costs (lengths) of its arcs.
- In the spanning tree problem, we need to determine a spanning tree of minimum cost (or length).

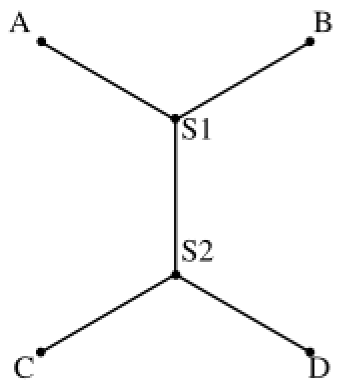
Steiner Tree Problem (1)

- Steiner Tree problem: Given a set *V* of points (vertices), interconnect them by a network of shortest length, where the length is the sum of the lengths of all edges.
- The difference between the Steiner tree problem and the minimum spanning tree problem is that, in the Steiner tree problem, extra intermediate vertices and edges may be added to the graph in order to reduce the length of the spanning tree.

Steiner Tree Problem (2)



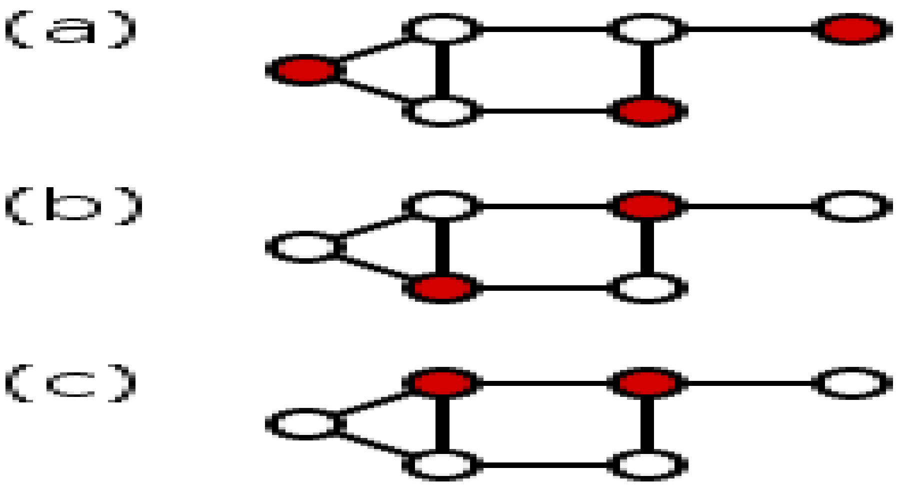
Steiner Tree for 3 points



Steiner Tree for 4 points

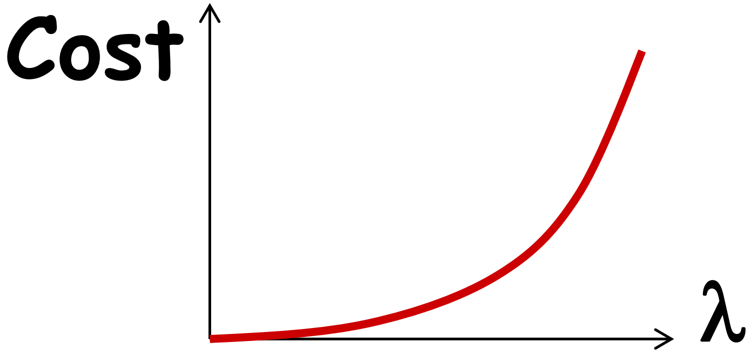
Minimum Dominating Set

For a graph G=(V,E), a **dominating set** is a subset *D* of *V* such that every vertex not in *D* is joined to at least one member of *D* by some edge.



Congestion Pricing Model

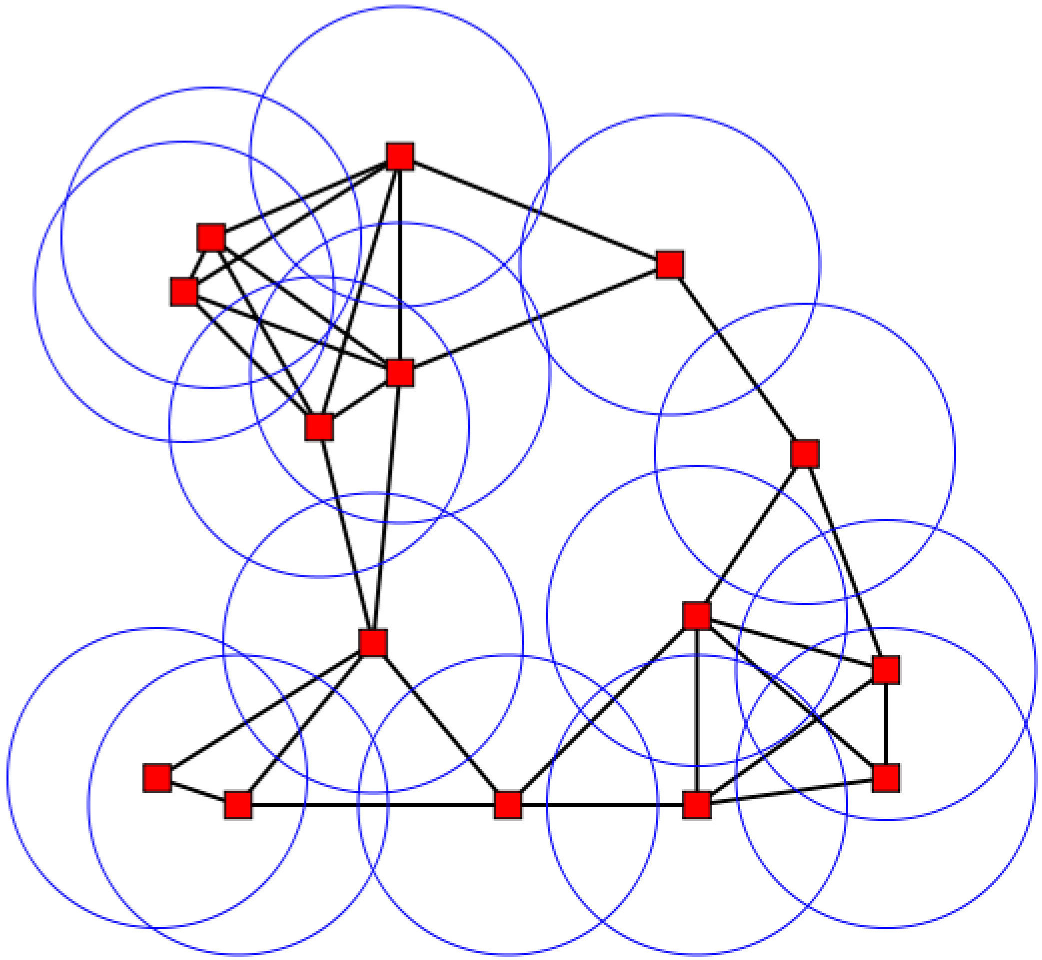
- Consider a resource of capacity C of which a fraction L has been consumed.
- The cost of the resource increases steeply as L approaches C (i.e. when the resource becomes a bottleneck)



Virtual Circuit Routing

Given a graph G = (V,E), a sequence of routing requests $\sigma = (s_1, t_1), (s_2, t_2), \ldots$ select a system of paths between s_i and t_i of bandwidth λ_i for each request so that the congestion (i.e. maximum load on any edge) is minimized

Position Based Routing in Dynamic Networks



A Unit Disk Graph Representation of an Ad hoc Network

Position Based Routing in Dynamic Networks

- Each node knows its position relative to its one hop neighbors in order to route packets
- Packets need to be forwarded in the geographical direction of the destination based on local information
- State changes and node movement can cause frequent changes in topology
- Not necessary to share state information globally since routing decisions can be done in a greedy manner based on only local information
- Routing decisions can be distributed and system will be scalable as long it is loop free.

References

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8. Images were obtained from the following web sites
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   3. http://www.timboucher.com/
   4. http://en.wikipedia.org/
   5. http://www.ee.surrey.ac.uk/
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